Unknown quantities…
If I asked you to say the first thing that comes to your mind about what Algebra is all about, I’ll bet many of you would say something about solving equations. Well, you’d basically be right!

One great way to describe Algebra is as the language of unknown quantities. That’s what equations are all about right? You are trying to figure out what values work for the equation.

What do we call the thing that represents those unknown quantities? It is a variable. We give the variable a letter to represent it…the most commonly used are x and y. So if we talked about variable x or variable y, we’d be talking about those unknown quantities (one or more numbers).

Today we are going to take a baby step towards working with equations. That baby step will be algebraic expressions.

What’s the diff???
So, you may be thinking to yourself, “equations, expressions…what in the world is the difference?” Well, given that mathematicians are very careful with words, there is most certainly a difference.

An expression is simply a collection of numbers, operations (+, -, *, ÷, etc.), grouping symbols (parenthesis), and variables. You can think of it as a very basic math sentence.

An equation throws in an equal sign. We’ll be getting to equations next lesson.

So whatcha do with an expression?
Simple…you replace the variable with numbers to find out the expression’s possible values. We call this evaluating the expression. Let’s try it out…

2(3 + 18 ÷ 9 – 7)  
hmm, where do we start? The answer you get depends on the order you do the math operations! How do we figure out the “right” way to solve this?

There is an old acronym that is great to memorize that helps me remember the order of operations. It is PEMDAS.

I remember it by Please Excuse My Dear Aunt Sally. Here is what it stands for:
Lesson 1-2: Algebraic Expressions and Models

- P – Parenthesis
- E – Exponents
- M – Multiplication (left to right)
- D – Division (left to right)
- A – Addition (left to right)
- S – Subtraction (left to right)

This is the order, P→S that we should do the math operations. OK, back to the problem.

2(3 + 18 ÷ 9 – 7)

We have parenthesis so we need to do the stuff inside there 1st.

OK, problem is we have a mixture of operations. PEMDAS says do ÷ then + then -. OK, here we go! 18 ÷ 9 = 2

2(3 + 18 ÷ 9 – 7) = 2(3 + 2 – 7) Now we can just go left to right doing the + and -.

= 2(5 – 7) Keep going…

= 2(-2) …and now finish off with the multiplication

= -4

This one had no variables, so we just get a number.

What happens if our expression has a variable?

If our expression has a variable, we will need to evaluate it for a specific value. In other words, we need to plug a number into it and evaluate it. Here is an example:

Evaluate the expression \((25a – 3) ÷ 8\) when \(a = 3\). The variable is \(a\), and we’re going to replace it with the number 3:

\[(25a – 3) ÷ 8 = (25 · 3 – 3) ÷ 8\] Replace the variable with the number 3

= (75 – 3) ÷ 8 PEMDAS: parenthesis 1st & multiply before subtract

= 72 ÷ 8 PEMDAS: now do the subtraction in the parenthesis

= 9 Finish off with the division
Another example

Evaluate $2t^2 - 3$ when $t = 4$. OK, wait a minute…we have a squared in there. What do we do with it? That “squared” is an exponent…a number or variable “raised to a power.”

Exponents / Powers

An exponent means “repeated multiplication.” For instance $2^5$ means 2 is used as a factor 5 times: $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$. There are five 2’s there. So multiply that out and you get 32.

You need to be careful with exponents. Consider the following:

Is $3x^5$ the same as $(3x)^5$? Nope. Let’s Evaluate both when $x = 2$.

The first: $3x^5$ means $3 \cdot x^5$ which means $3 \cdot 2^5 = 3 \cdot 32 = 96$.

The second: $(3x)^5$ … PEMDAS says inside parenthesis first…

$$(3x)^5 = (3 \cdot 2)^5 = (6)^5 = 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 7,776.$$  

So, huge difference…96 and 7776.

The exponent only affects the thing immediately to the left of it. If you want it to work on more than one thing, you need to put all that stuff inside parenthesis. Check it out…

Evaluate the following when $x = 2$. They look similar so be careful!!!

$$(3x + 7)^3 = (3 \cdot 2 + 7)^3$$  
Replace the variable with the number…

$$(3 \cdot 2 + 7)^3$$  
PEMDAS: inside parenthesis first…multiply

$$(6 + 7)^3$$  
PEMDAS: now add (inside parenthesis…)

$$(13)^3$$  
PEMDAS: now the exponent…$13 \cdot 13 \cdot 13$

$$= 2,197$$  
😊

$$(3x)^3 + 7 = (3 \cdot 2)^3 + 7$$  
Replace the variable with the number…

$$(6)^3 + 7$$  
PEMDAS: inside parenthesis first…multiply

$$= 216 + 7$$  
PEMDAS: exponent next…$6 \cdot 6 \cdot 6 = 216$

$$= 223$$  
😊
Lesson 1-2: Algebraic Expressions and Models

\[ 3x^3 + 7 = 3 \cdot 2^3 + 7 \]
Replace the variable with the number...
\[ = 3 \cdot 8 + 7 \]
PEMDAS: exponent next... \(2 \cdot 2 \cdot 2 = 8\)
\[ = 24 + 7 \]
PEMDAS: multiplication before addition
\[ = 31 \]

So, back to that example: Evaluate \(2t^2 - 3\) when \(t = 4\)

\[ 2t^2 - 3 = 2 \cdot 4^2 - 3 \]
Replace the variable with the number...
\[ = 2 \cdot 16 - 3 \]
PEMDAS: exponent first... \(4^2 = 16\)
\[ = 32 - 3 \]
PEMDAS: multiplication before subtraction
\[ = 29 \]